APH Meeting on

Needs of Blind Students in Mathematics

held in

Louisville, Kentucky

July 6-8, 1979

Frank L. Franks, Editor
Department of Educational Research
American Printing House for the Blind

Needs Meeting Organizers

Frank L. Franks
Research Scientist
Department of Educational Research
American Printing House for the Blind
Louisville, Kentucky

June E. Morris, Director
Department of Educational Research
American Printing House for the Blind
Louisville, Kentucky

Participants

Tony Evancic
Philadelphia City Schools
Philadelphia, Pennsylvania

Sandra Albrecht
Florida School for the Blind
St. Augustine, Florida

Stephanie Richards
Indiana School for the Blind
Indianapolis, Indiana

Carol Jackson
Steele Elementary School
Colorado Springs, Colorado

Tom Ridgeway
Georgia Academy for the Blind
Macon, Georgia

Margaret Ritchie
Pasadena Unified School District
Pasadena, California

Tuck Tinsley III
Florida School for the Blind
St. Augustine, Florida

Marsha Williams
Vista Unified School District
Vista, California

Sue Young
Tennessee School for the Blind
Nashville, Tennessee

Gregory Holmes
University of Louisville
Louisville, Kentucky

A national meeting in mathematics needs of blind students was held in Louisville, July 6-8, 1979. Ten expert teachers of visually handicapped students in day and residential school programs participated in the sessions. Teachers included secondary mathematics teachers of blind students, elementary grade teachers of mathematics, resource teachers, and itinerant teachers who teach and assist visually handicapped students, and primary and early childhood teachers who work with visually handicapped students. One blind college student who attended public school was included to present the student point of view. Two of the participants are blind mathematics teachers.

The purpose of the meeting was to identify specific mathematics aids and materials that will be needed by visually handicapped students in near future and to prioritize the needs for such materials. Consideration was given to the direction in which mathematics curricula are headed and to new emphasis in mathematics instruction.

I. Mathematics Content Assessment by Experts

Participants were asked to consider mathematics needs of visually handicapped students in terms of basic mathematics concepts/operations as assessed by six national mathematics content experts, after they had the opportunity of assessing the items themselves. The mathematics content experts who assessed the items are:

Dr. Grayson H. Wheatley, Professor
Mathematical Sciences Building
Purdue University
West Lafayette, Indiana 47907

Dr. E. Glenadine Gibb
Mathematics Education Department
RLF 8-100
The University of Texas at Austin
Austin, Texas 78712

Dr. Evelyn M. Neufeld
Associate Professor
School of Education
San Jose State University
San Jose, California 95192

Dr. Jack Price, Superintendent
Vista Unified School District
Vista, California 95192

Mr. George Immerzeel
Price Laboratory School
University of Northern Iowa
Cedar Falls, Iowa 50613

Dr. Arthur Kessner
Lawrence Hall of Science
University of California
Berkeley, California 94720

Content experts assessed mathematical concepts and operations to identify those which are critical to performance in and understanding of mathematics processes. They were provided the following information and instructions:

The American Printing House for the Blind (APH) is in the process of identifying needs in mathematics for blind and visually handicapped (vh) students. Much information, particularly in elementary and junior high mathematics texts, is presented or introduced pictorially/schematically and is of extremely limited value to the blind and vh student. When this information underlies the introduction and/or comprehension of basic mathematics concepts, there is a need to develop tactile aids and/or alternative procedures for presenting the conceptual information to these students.

We want you to respond to the items in this assessment to provide us with preliminary information for identifying critical concept areas. This information will be used by mathematics teachers of blind and vh students in setting priorities for APH materials development projects in mathematics. Your responses will provide a content base for the specialist teacher of blind and vh students in mathematics to work from. The specialist teacher will:

1) Identify units or/concept areas where learning for the blind and vh student can be facilitated/improved with the provision of tactile aids and materials,

2) Prioritize these, and

3) Draft specifications for high priority aids and materials identified.

A copy of the assessment identifying highest priority items is appended to this report.

Content experts were asked to mention manipulative/tactile aids they might be aware of which have implications for introducing/illustrating/demonstrating concepts to blind students. Experts also were asked to anticipate new devices (e.g., teaching machines) and major emphases (e.g., probability and statistics) which may appear in or have impact on the mathematics curriculum in the next 3 to 5 years.

II. Priority Needs Established by Teachers of Visually Handicapped

The participants met in general sessions, broke into group sessions (k-6 and 7-12), and met again in a final general session. Considerable time was spent in discussion of mathematics instruction, with teachers citing individual problems and solutions. In the group sessions, there was greater focus on specific aids and materials. Each group prioritized needs, and in the final session, reviewed each others priorities in setting overall materials needs.

The highest priority established was for the development of entry level, primary grade math materials which are designed to introduce/illustrate/teach concretely basic math operations and concepts to blind students. Early math texts are too visual and cannot be transcribed into braille with meaningful tactual representations. Consensus was unanimous for these materials.

Additional high priority materials included: a teacher's guide to instruct students in a systematic approach to examining graphs and charts, investigation and evaluation of commercially-available materials by APH to determine those which can be used or adapted by visually handicapped students, more two- and three-dimensional geometry materials, a tactually legible meter stick for blind students, metric area measurement materials, adaptation into braille of the Key Math Test (American Guidance Service), item analysis of SAT in math, more calculator materials--and for younger students, development of a geometric re-source book of plane figures, improvement of the quality of shading and graphics

in all areas of braille reproduction of algebraic and geometric illustrations, need for a math educator who will proof all plates prior to printing in math subject matter, and simplification of Nemeth code for basic school texts.

A number of aids were discussed but were dropped when it was determined that alternative methods and instruction could be used to teach the concepts involved.

Time limitations restricted the detail supplied on high priority items. Participants were extremely interested in entry-level and primary grade materials. Consequently, they spent most of the available time in this area. Their report follows.

<div align="center">

APH National Math Needs Meeting

Primary Math Project

</div>

<u>Preliminary Proposal</u>

Recognizing the unique needs of the young blind student and the difficulties they must overcome to compete in the area of math due to their special needs, the following proposal is presented.

APH will undertake to develop, design, and package learning materials that address themselves to meet these special needs. That these materials be flexible enough to be used by all teaching resource persons involved in the education of the blind such as itinerant, resource room teachers, regular classroom teachers, and self-contained classes for the visually handicapped.

We have included skills and concepts of great importance to the young blind child that must be considered beginning entry level through grade six. For con-venience we have divided the project into three sections: Pre-kindergarten or readiness, primary, and intermediate. All materials are for use with multi-handicapped students.

I. APH will develop and package "Basic Math Manipulative Skill Kits" for the primary through upper elementary grades. (pre-kindergarten-6th grade)

Each kit will contain tactile aids and manipulatives designed to introduce/ illustrate/demonstrate/teach concretely basic math operations and concepts to the blind student.

Kits will be designed for multipurpose usage throughout all elementary grades and beyond, with the following basic entry levels:

     a. pre-kindergarten
     b. primary 1-3
     c. upper elementary 4-6

Kits will be carefully coded or named so as to facilitate use through all grade levels.

II. Kits will contain:

1. Manual of concrete/task oriented activities
2. Packaged manipulative objects--beads, pegs, sticks, rods, shapes, etc. (according to entry level)
3. A behavioral objective check list for student evaluation and assessment.
4. Cross reference of materials and skills chart.

Designated Priorities

1. All concepts/skills, beginning with prenumber concepts will be presented in a developmental sequence.
2. All APH current math learning materials will be evaluated for use.
3. Commercial learning materials will be evaluated for adaptation and possible use.
4. APH to design and develop or adapt any learning materials (aids) as needed to fill specific content areas not covered.
5. Possible use of cassette taped activities to be considered.
6. Cross reference of materials and skills chart to be developed.

Suggestions for Materials for Primary Math Project

Kit to contain: (These may or may not need adaptation.)

1. "objects" for association-making and understanding relationships, etc.

2. Beads of different shape and size for stringing patterns

3. Pegs of different shape/size and peg board

4. Number concept cards (make tactile at APH). ETA $3.75

5. Unit blocks 1-10 from Houghton-Mifflin Co.

6. Counting Board from Houghton-Mifflin Co.

7. Unifix 1-10 stairs from ETA $3.00

8. Value Boats 1-10 from ETA $4.50

9. Adapt ETA's apple fraction set (see blue sheet)

10. Number Concept Cards 1-10 (adapt in braille) from ETA #4050 $3.75

11. Judy #750 Fraction Inlay Board--? cost

12. APH numberline adapt for missing numerals and sequencing higher numbers

13. APH Shape Board

14. "Piggy Banks"--in print and braille money values (pennies, nickels, dimes) 1¢-25¢

15. Geared clock with numerals in print and braille

16. Place Value Teaching aid for hundreds, tens, and ones (see blue sheet)

17. Addition, subtraction, multiplication, and division flashcards in print and braille (ID 7786-ID 7789)

18. Attribute Games from Teaching Resources or Desk Top Attribute Games by Invicta Educational Aids (I.D. 1277)

19. Variform Inset Placing Trays #123D from ETA

20. Graduated colored cylinders from Teaching Resources, 100 Boyston Street

21. Place Value--Funda math from Ideal (ID 775--all Florida School Supply Cat.)

22. Ideal Geoboard for geometry (ID 7540--All Florida)

23. Same and different cards--Sandy's class

24. "Missing addends" cards--Sandy's class

25. "Operations" game for addition and subtraction--see Sandy's class

26. More materials will be considered for inclusion in kit at a later date

A breakdown of specific aids and relevant comments from the meeting follows.

Calculator materials

Adapt Texas Instruments' "Little Professor" and "Quiz Kit" for a voice-output for independent, self-instruction and practice in basic mathematics

operations. APH should explore this adaptation with Texas Instruments and also explore the possibilities of adding speech to the ABLE calculator for use with primary students.

Develop materials to _introduce_ the calculator to primary-elementary grade students.

Develop and evaluate specialized aids, including overlays or shields, for learning the keyboard and key operations to facilitate beginning student use of the calculator.

Develop/produce primary, elementary, and secondary problem decks of activities in large print and braille for the calculator, such as the Iowa Problem Solving cards.

Develop more advanced materials which emphasize problem solving for older students.

Mathematics--testing

Analyze SAT to identify areas of need and deficiency in mathematics to facilitate itemizing mathematics needs in the future. Look at the tests to determine if there is poor problem representation which can result in poor performance by the blind student. Publish the results in the AEVH journal.

Adapt and put the key   ih Test (American Guidance Service) into braille. Use the same format as in print if possible.

Metrics

Develop/produce a legible meterstick in braille, with specifications approved at 1977 Annual Meeting. (The Howe Press model is too busy, too complicated.)

Develop/produce a metric area measurement kit following the format of the readiness (metrics) kits.

Develop/produce a metric measuring kit to introduce metric measurements, concepts, and skills.

Fraction understanding--entry level

Use a take-apart apple or other three-dimensional fruit with fractional values to show that a whole object can be divided into halves, fourths as beginning to fractional terms of "whole," "half," and "fourth."

Money understanding--entry level

Real money can be used, but a manual is needed.

Place value to hundreds

An aid for teaching place value with ones, tens, hundreds by relating the quantity to numerals. The aid can be composed of three sections and three columns. Popsicle sticks can be used to match the numerical values shown on the display.

Manipulative materials--K-3

Development and provision of motivational and high interest manipulatives for introducing mathematics concepts.

Basic mthematics operations--facts to 9

Consider adding speech to "Charlie, the Robot"...

Sequence of numbers--primary grade level

Develop a math board on which the only number pieces that fill into a given strip are the correct answers for the problems on that strip.

Additions and inequalities--entry level

The aid consists of blocks marked with braille and large print numerals according to the number of units in each block. Each block is grooved according to the number of units it represents, so that the student can feel each separate unit contained in the block.

Geometry

A geometric resource book for plane figures is needed. It should be:

1) Thermoformed
2) Loose-leafed
3) One illustration per page

4) Large enough to measure parts
        5) Grouped by figures--first without, and then with associated parts
        6) Supplied with table of contents

2- and 3-D Geometry

        Develop additional 2- and 3-D materials.

APPENDIX A

Mathematics Content Assessment

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fying needs in mathematics for blind and visually handicapped (vh) students.
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is presented or introduced pictorially/schematically and is of extremely limited
value to the blind and vh student. When this information underlies the intro-
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develop tactile aids and/or alternative procedures for presenting the conceptual
information to these students.

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preliminary information for identifying critical concept areas. This informa-
tion will be used by mathematics teachers of blind and vh students in setting
priorities for APH materials development projects in mathematics. Your responses
will provide a content base for the specialist teacher of blind and vh students
in mathematics to work from. The specialist teacher will:

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student can be facilitated/improved with the provision of tactile aids and
materials,

2) Prioritize these, and

3) Draft specifications for high priority aids and materials identified.
While few blind and vh students have been enrolled in advanced high school mathe-
matics courses, we do want input in these areas. Recently, efforts have been
made to develop an aid for introducing two- and three-dimensional spatial re-
lationships in mathematics to blind students. Are you aware of such manipula-
tive/tactile aids for introducing/illustrating/demonstrating concepts to sighted
students which have possible implications for use by blind students, perhaps
with adaptation?

Also we want to anticipate new devices (e.g., teaching machines) and major emphases (e.g., statistics and probability) which may appear in the mathematics curriculum in the next 3 to 5 years in an effort to develop appropriate materials by the time vh students need them.

Please use the following rating scale:

1--Priority 1: Highest priority. The item identifies basic information or information which underlies learning and/or comprehension of critical concepts in mathematics.

2--Priority 2: Integral information. The item identifies important conceptual information which is an integral part of and/or is recurring information in the mathematics curriculum.

3--Priority 3: General information. The item identifies information which may be desirable, but is not critical to learning or comprehending basic or critical mathematics concepts.

4--Priority 4: Low priority. The item identifies information of a lower priority than the above three categories.

The blank to the left of each item provides space for your response. There is a blank for each unit or area and a blank for each item or component. Please assign a priority number to each category and then prioritize each item within each category.

You may add items and/or sections and items. These may be assessed in the same way as those appearing in the assessment. If you wish to describe or comment on an item or aid, please do so.

Since items appear in four categories in the assessment, there may be some overlap.

The I items are clearly
the highest
priorities designat...
buy the Content Review...
Checks (✓) indicate more
I's than other responses

I. MATH CONCEPTS--NUMBER SYSTEMS AND OPERATIONS--GRADES K-6

A. Kindergarten: Concepts/Tasks/Operations

__I+__ 1. Manipulation of sets of concrete objects

__'__ 2. Members and non-members of sets -only with a great variety of concrete materials

__✓__ 3. 1-1 correspondence -only by matching various sets of things like cups/saucers; objects that would usually be found in a playhouse

__ __ 4. Conservation of number -usually not developed by the child until 7 or 8

__I+__ 5. Classifying objects

__I__ 6. Comparison of sets (more or less) -only gross, part-to-part comparisons of groups of objects

__✓__ 7. One more and one less than -only in connection with physical objects

__✓__ 8. Ordering of sets and numbers (using number lines) -save for third grade

__I+__ 9. Counting orally 1-10 -only connected to counting objects

__I__ 10. Reading numerals 0-9 -only in connection with recording constructed numbers

__✓__ 11. 1-1 correspondence (number to element of set) -save for later grade level

__ __ 12. Writing numerals 0-9 -only in connection with recording number construction

__I+__ 13. Constructing a set for a given number

__ __ 14. Comparing numbers 0-9 -save for later grade level

__ __ 15. Ordinal numbers 1st-5th -only verbally, related to block constructions or other three-dimensional objects

__ __ 16. Recognizing grouping by 10's -only with blocks

__ __ 17. Introducing number concepts 10-19 (teens) -save for third grade

__ __ 18. Grouping by tens where there is a non-zero numeral in the 1st column -with blocks

__ __ 19. Identifying how many tens, how many ones when given a two digit numeral -only wh... number has been constructed with blocks

__ __ 20. Reading and writing numerals 0-99 -but only in recording constructed numbers on recording sheets

__ __ 21. Introduction of ordering of numbers -save for later grade

__ __ 22. Introduction of the concept of odd and even numbers 0-30

__ __ 23. The >,< relationship between numbers 0-99

__ __ 24. Writing number sentences from picture problems

__ __ 25. Place value

__ __ 26. Fractional number 1/2 -but provide physical experiences with halves of fruit, vegetables, cups of sand, packages of nuts, cups of liquid, etc.

A. Kindergarten: Concepts/Tasks/Operations (Con't.)

_ _27. Union of sets (sets) of exponential blocks

● _ _28. Introduction of symbols +,- –*only as it naturally is used in addition and* *subtraction of number construction*

_ _29. Solving equations to sums of 5 *-too abstract*

_ _30. Using the number line for addition *-number lines are very abstract*

_ _31. Subtraction grouping into sets *-only with exponential blocks*

_ _32. Solving subtraction equations with numerals of 5 or less *-only with exp. block.*

_ _33. Using the number line for solving subtraction problems   *Save for later, altho some children will*

_ _34. Inverse relationship between addition and subtraction   *grasp these concepts with the appropriate*

_ _35. Commutative property of addition   *preliminary experienc with block constructi*

_ _36. Sums and differences, xxxxxxxxxxx *-with exponential* *blocks no need to limit in this way*

_ _37. Vertical xxxxxxxxxx notation + and - *-save horizontal notation for later*

_ _38. Cooking where children cook with *little intervention from adults*

_ _39. *Simple inventories, tallies where physical objects are used*

● _ _40. *Simple graphs using physical objects*

_ _41. *Simple surveys where classmates, teachers, family members are questioned about* *favorite books, foods, colors, birthdays, etc.*

_ _42. *Games; especially games involving classification and seriation*

B Grade Level 1: Concepts/Tasks/Operations

_/_1. Comparison of sets and number of objects in sets (1-1)

_ _2. Inequalities: greater than, less than *-only in relation to towers of blocks*

_✓_3. Cardinal numbers to 500 *-only in recording number constructions*

_/_4. Place value to ~~hundreds~~ *thousands*, using bundles of ten *--using exponential blocks*

_✓_5. Reading and writing numerals to 500 *-only in connection with constructing and* *recording numbers*

_ _6. Using various names for same number

_✓_7. Equation as two names for same number

_✓_8. Fractional parts of objects 1/2, 1/3, 1/4

● _✓_9. Reading numerals 1/2, 1/3, 1/4

_✓10. Ordering to 500

_/11. Before, after, between, one more, one less

B. Grade Level 1: Concepts/Tasks/Operations (Con't.)

__/ 12. Introduction and use of ✗✗✗, and = symbols *only in relation to recording of block constructions*

__/ 13. Counting by twos, fives, and tens *only with blocks*

__ 14. Number line

__ 15. Expanded notation

__✔ 16. Ordinal numbers 1st through 10th *in dealing with physical objects, people, etc.*

__/ 17. Comparing numbers $>$,$<$ *only with towers of blocks*

__✔ 18. Writing number sentences from picture problems

__ 19. Estimation *only in the most casual way with objects*

__/ 20. Addition equations sums 10 or less

__ 21. Commutative property of addition via computational skills *only when noticed in addition block constructions*

__/ 22. Subtraction ✗✗✗✗✗✗✗✗ *with exponential blocks no need to limit*

__/ 23. Sums and differences ✗✗✗ *with exponential blocks no need to limit*

__ 24. Inverse relationships *only when noticed in relation to block construction*

__ 25. Addition: ✗✗✗✗✗✗ *with exponential blocks no need to limit*

__ 26. Associative property of addition via computational skills *only in relation to block construction*

__ 27. Expanded notation: sums 11-18

__ 28. Vertical notation: ✗✗✗✗✗✗

__ 30. Column addition

__ 31. 2 digit addition

__ 32. 2 digit subtraction   ALL WITH EXPONENTIAL BLOCKS

__ 33. 3 digit addition

__ 34. 3 digit subtraction

__ 35. Cooking

__ 36. Graphing

__ 37. Classification/Seriation tasks with attribute blocks and other objects

__ 38. Games

__ 39. Puzzles

__ 40. Inventories/Surveys

C. Grade Level 2: Concepts/Tasks/Operations

_/_1. Review place value (1's, 10's, 100's)

_/_2. *Review* ~~Introducing~~ place value (1000's)

_/_3. Comparison of numbers, 0-1000 -*using block constructions*

_ _4. Review of sets-*this would be an appropriate time to introduce sets*

_/_5. Cardinal numbers to 1000 -*only in relation to recording number constructions*
    unless a stable foundation in this has been laid earlier

_ _6. Ordinal numbers from 1st to 50th

_/_7. Reading numerals to 1,000 -*only after adequate experience with constructing and*
    recording

_✓_8. Writing numerals to 1,000 -*not merely by rote. This is unnecessary if children*
    have had adequte experience since kindergarten in constructing and recording

_/_9. Ordering to hundreds -*although the earlier place value tasks might make this*
    unnecessary

_ _10. Fractions on the number line

_ _11. Fractional numbers 1/2, 1/3, 1/4, 2/3, 3/4

_ _12. Fractional number 1/2

_ _13. Writing numerals to 1,000 in expanded notation

_ _14. Odd and even numbers -*use exponential blocks*

_/_15. Addition and subtraction fact through ~~18~~ -*with exponential blocks children*
    should have no problems with addition and subtraction through to thousands place

_ _16. Commutative and associative properties of addition -*this is an appropriate time*
    to emphasize these properties in connection with their block constructions and ac

_ _17. Adding ~~two~~ digit numbers with sums ~~less than x,000~~ -*do not limit*

_ _18. Subtraction ~~of two digit numbers~~ -*do not limit*

_✓_19. Column addition -*excellent task; be sure to capitalize by making comparisons whic*
    lead directly to seeing the associative property

_ _20. Regrouping (borrowing, carrying) using 3 digits carrying to 10's or 100's
    column, borrowing to 10's or 100's column

_ _21. Continued practice with basic addition and ~~100~~ subtraction ~~facts~~

_ _22. Introduction to multiplication ~~(sets x number x line)~~ repeated addition

_ _23. Multiplying by 0,1, etc.

_ _24. Introduction to 100 basic multiplication facts -*memorization of facts should*
    be introduced to those students who have a thorough grouding in the concepts

_ _25. Commutative, associative and distributive properties of multiplication

_ _26. Introduction to division ~~(by set x number x line~~ repeated subtraction)

_ _27. Multiplying by multiples of 10 and 100

C. Grade Level 2: Concepts/Tasks/Operations (Con't.)

___ _28. Ordered pairs

● _29. *Square numbers with blocks*

___ _30. *Continue with place value recording sheets for those in need of them*

___ _31. *Games, especially those involving logical thinking*

___ _32. *Graphing, surveys, inventories. Emphasis now on tabulations and recording*

___ ·33. *Cooking; inventing recipes*

D. Grade Level 3: Concepts/Tasks/Operations

_/_±1. Place value for numbers less than 1,000,000

_/_2. Cardinal numbers to millions

_/_3. · Whole numbers

___ _4. Fractions 1/5, 1/6, 1/8, 1/12

_✔_5. Numeration to millions

_/_6. Regrouping through hundreds

● __7. Roman numerals

___ _8. Equivalent fractions

___ _9. Ordering with numbers to millions

___ _10. Ordering with fractional numbers

_✔_11. Vertical x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶forms of multiplication x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶

_/_12. Basic addition and subtraction facts

___ _13.. Order (commutative) and grouping (associative)

_✔_14. Column addition -- use vertical x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶

_/_15. Subtraction -- regrouping hundreds and thousands (borrowing, using zeros)

___ _16. Multiplication algorithm

_✔_17. Developing competency using basic 100 multiplication facts

_✔_18. Working with number line

●/_19. Multiplication by multiples of 10, 100, 1000

_/_20. Introduction to division (x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶x̶) -only with exponential blocks

D. Grade Level 3: Concepts/Tasks/Operations (Con't.)

___21. Multiplication and dividing to 10's and 100's

___22. Recognizing pairs of numbers where one number is 100 times another

___23. Estimating to nearest 10, 100, 1000

___24. Multiplying -- Adding (distributive) principle

___25. Multiplying multipliers less than 1,000 using partial products

___26. Multiplication algorithm

___27. Multiplication by 10's, 100's,.1000's

___28. Dividing by repeated subtraction "sideline method" 1 digit division

___29. Dividing by 10's dividing by numbers between 10 and 100

___30. *Games that require logical thought: e.g. Mastermind, Foresight, chess, checkers,*

___31. *Logic Puzzles*

___32. *Measurement activities involving area, length, and volume but only with exponential blocks in the metric measurement*

___33.

___34.

___35.

I. MATH CONCEPTS--NUMBER SYSTEMS AND OPERATIONS--GRADES K-6 (Con't.)

. Grade Level 4: Concepts/Tasks/Operations

_✓_ 1. Place value for numbers less than 1,000,000,000 -*only as review if previously recommended sequence has been followed*

_✓_ 2. Reading and writing large numbers - *only as review*

_✓_ 3. Expanded notation

_/_ 4. Whole numbers

_/≠_ 5. Order relations (using $<$, $>$, $=$, $\neq$)

_/_ 6. Number patterns

___ 7. Common multiples

___ 8. Common factors

___ 9. Prime numbers

___ 10. Equivalent fractions (includes reducing to lowest terms) -*save for later grade*

___ 11. Relationships between two fractions (greater than, less than, equal)

___ 12. Improper fractions and mixed numbers

_/_ 13. Introductions to decimals

___ 14. Numerals in base 5

_/_ 15. Computations in four basic operations -*review*

___ 16. Review association and commutative property for addition and multiplication

_/≠_ 17. Multiplying by 10, 100, and 1,000

_✓_ 18. Column addition with addends less than 10,000

_✓_ 19. Subtraction of numbers less than 10,000

_✓_ 20. Multiplication with multipliers less than 1,000

_/_ 21. Division algorithm (one digit division with remainder)

___ 22. Averages

_✓_ 23. Recognize pairs of numbers where one is 10x other

___ 24. Division by multiples of 10

___ 25. Division -- 2 digit divisor with remainder

_✓_ 26. Addition of like factors

E. Grade Level 4: Concepts/Tasks/Operations (Con't.)

_27. Subtraction of like fractions

_28. Multiplication of fractions and division of fractions by whole numbers

_29. Exponents

/30. Number sentences solving story problems

_31. *All tasks that have been done with blocks and other objects should now*
be done at the pictorial or verbal level. New concepts should, however,
_32. *still be done with blocks and objects.*

_33. *Bases other than ten-with the appropriate blocks*

_34. *Logic games and puzzles*

_35. *More formal graphs, surveys and mappings*

F. Grade Level 5: Concepts/Tasks/Operations

/1. Review of place value -*Probably unnecessary if my sequence is followed*

✓2. Number patterns and sequences

✓3. Extended use of place value

✓4. Expanded decimal notations

_5. Equivalent fractions and decimal numerals

/6. Estimating with decimals (whole, tenths, hundredths)

_7. Reducing fractions

_8. Improper fractions as mixed numbers

_9. Mixed numerals as improper fractions

_10. Order relation with fractional numbers

_11. Reciprocals and applications

_12. Ratios

/13. Decimal fractions

/14. Decimal fractions hundredths

_15. Base 3, 4, 5 numeration

/16. Prime numbers

✓17. Prime factorization

F. Grade Level 5: Concepts/Tasks/Operations (Con't.)

/ 18. Set of multiples

/ 19. Least common multiples

/ 20. Sets of factors

/ 21. Greatest common factors

/⁺ 22. Review for mastery of basic facts

/ 23. Computation in four basic processes including word problems, practice
estimations of answers

___ 24. Meaning of exponents

___ 25. Basic fact in non-decimal numerations systems

___ 26. Addition and subtraction of unlike fractions

___ 27. Multiplication of fractions

___ 28. Division of whole numbers with remainders (2 digit divisors)

/ 29. Addition, subtraction of decimal numbers

___ 30. Literal variables in number sentences

___ 31. Introductions to per cent

___ 32. Using exponents

___ 33. Continue with formalization of all earlier concrete experiences

___ 34. Begin concrete experiences with concepts such as fractions, probability,
measurement, proportions

___ 35. Continue with logic games, including Wff-n-Proof

___ 36.

___ 37.

G. Grade Level 6: Concepts/Tasks/Operations

/⁺ 1. Order relations: whole numbers

/ 2. Fractional part

/⁺ 3. Equivalent fractions

/⁺ 4. Reducing fractions

/ 5. Fractional numbers > 1

___ 6. Improper fractions as mixed numeral

G. Grade Level 6: Concepts/Tasks/Operations (Con't.)

_____ 7. Mixed numerals as improper fractions

✓ 8. Order relations, fractional number

_____ 9. Reciprocals

✓ 10. Fractional number as decimal fractions

✓ 11. Decimals as proper fractions

_____ 12. Renaming decimal fractions

/ 13. Per cent

_____ 14. Non-decimal numeration: base 2

/ 15. Prime numbers

/ 16. Prime factorization: factor trees

/ 17. Set of factors

/ 18. Greatest common factor

/ 19. Sets of multiple

/ 20. Least common multiples

/ 21. Review of addition, subtraction, multiplication and division of whole numbers

_____ 22. Dividing using division algorithm, with and without remainders, 2 and 3 digit divisors

_____ 23. Fractions: indicated division

✓ 24. Addition and subtraction of fractional numbers with and without like denominat

/ 25. Addition and subtraction of decimal numbers

/ 26. Multiplication and division of decimal numbers

_____ 27. Basic facts in non-decimal numeration systems

_____ 28. Computing square roots of perfect squares

_____ 29. Addition and subtraction -- percent notation

_____ 30. Adding and subtracting positive and negative integers

_____ 31. Continue with formalization of earlier concrete concepts

_____ 32. Continue with logic games and puzzles

_____ 33. Continue with proportions, fractions, estimation, logical thought problems

_____ 34. Introduce algebra (addition and subtraction) with exponential blocks

II. ADDITIONAL CONCEPTS/CATEGORIES IN MATH

___ **A.** Measurement (metric section follows)

    / 1. Measuring and measurement

    / 2. Standard units

    / 3. Linear measurement

    / 4. Area measurement

    / 5. Volume/capacity measurement

    / 6. Mass/weight measurement

    ____ 7. Measuring and drawing angles

    ____ 8. Finding "missing" sides in geometric shapes

    ____ 9.

    ____ 10.

    ____ 11.

Primary Grades: these concepts should be experienced only in terms of "how many" and in relation to real objects. For example, volume could be measured by filling a box with cubes in order to determine how many it takes.

Upper Elementary Grades: building on the active experiences at the earl[y] grade level, measurement experience[s] can now be more formalized.

___ **B.** Metric Measurement

    / 1. Measuring without numbers

    / 2. Measuring with numbers

    / 3. Prenumber classifying, matching, ordering

    / 4. Comparisons by iteration

    / 5. Standard units

    / 6. Working with standard units

    / 7. Measuring with standard metric units

    / 8. Metric linear measurement principles

    / 9. Metric areal measurement principles

    / 10. Metric volume/capacity measurement principles

    / 11. Metric mass/weight measurement principles

    ____ 12.

    ____ 13.

    ____ 14.

SEE ABOVE

II. ADDITIONAL CONCEPTS/CATEGORIES IN MATH (Con't.)

_____ C. Geometry

     / 1. Identification of basic shapes

     / 2. Nonmetric concepts (i.e. points, lines, planes, angles, etc.)

     / 3. Two-dimensional spatial relationships

     / 4. Three-dimensional spatial relationships

     _____ 5. Properties of parallel lines

     _____ 6. Properties of perpendicular lines

     / 7. Identifying symmetrical shapes

     _____ 8. Identifying congruent shapes

     _____ 9. Identifying similar shapes

     _____ 10. Angle sums in triangles and polygons

     _____ 11. Theorem of Pythagoras

     _____ 12. Trigonometric ratios

     / 13. Properties of the circle

     _____ 14. Properties of spheres, cylinders, and cones

     _____ 15. Three-dimensional/solid geometry

     _____ 16.

     _____ 17.

     _____ 18.

     _____ 19.

_____ D. Number Theory

     _____ 1. Subset of whole numbers

     / 2. Factors

     _____ 3. Prime and composit numbers

     _____ 4. Divisibility

     _____ 5. Union and intersection of sets

     _____ 6. Equations and inequalities

     / 7. Positive and negative rational numbers

Primary Grades: children should experience geometry in active situations where shapes are physically handled, moved from place to place, rotated and flipped in various ways, and generally they are involved in conversations about what they are experiencing.

Upper Elementary: children should build on the active experiences of the earlier period and begin formalize their experiences. Children at this level are, however, still quite concrete in their thinking and this should be kept in mind. Only at the end of elementary school will the more abstract concepts listed here be meaningful and then only if introduced with physical objects. Of course, this refers to the average and it is true that a certain percentage of children are capapble of abstract work early

SAVE FOR UPPER ELEMENTARY

II. ADDITIONAL CONCEPTS/CATEGORIES IN MATH (Con't.)

D. Number Theory (Con't)

_____ 8. Absolute value

_____ 9. Algebraic expressions and reasoning

_____ 10. Functions and rational numbers

_____ 11. Geometric numbers and roots

_____ 12. Irrational numbers

SAVE FOR THE END OF THE UPPER ELEMENTARY YEARS--(be prepared to eliminate if a particular group of children cannot grasp since this is very abstract)

_____ 13. Simplifying expressions to find value

_____ 14.

_____ 15.

_____ 16.

_____ 17.

E. Tables, Charts and Graphs

/ 1. Interpreting a picture graph

/ 2. Interpreting bar, circle or line graphs

/ 3. Interpreting tables or charts

/ 4. Finding points and graphing ordered pairs

If children have had adequate experience with earlier graphing, surveys, and inventories, upper elementary children could begin to use tables, charts, and graphs in the more formal ways listed here.

_____ 5. Graphing solutions of inequalities and equations

_____ 6. Rotations and translations

_____ 7. Enlarging figures

_____ 8. Graphing integer functions

/ 9. Graphing statistical data

_____ 10.

_____ 11.

_____ 12.

_____ 13.

F. Other Mathematical Systems

_____ 1. Binary system

_____ 2. Base 12 system

_____ 3.

With the appropriate blocks, other mathematical systems are appropriate experiences for primary children. Even first grade children are able to handle constructing, recording, and addition and subtraction in other bases. sure they have the base ten system with block

_____1. Algebra and Trigonometry

_____1.   Working with basic formulas

_____2.   Working with polynomials and functions; factoring

_____3.   Radicals and irrational numbers

_____4.   Quadratic equations and functions

_____5.   Quadratic relations and systems

_____6.   Exponential functions and logarithms

_____7.   Sequences and series

_____8.   Matrices

_____9.   Trigonometric functions

_____10. Circular functions

_____11. Inverse functions

_____12.

_____13.

_____14.

_____15.

SAVE FOR UPPER ELEMENTARY GRADES (introduce with physical objects) AND CONTINUE MORE FORMALLY AT THE JUNIOR HIGH AND HIGH SCHOOL LEVELS

III. SPECIFIC INSTRUCTIONAL UNITS/KITS OF MATERIALS IN MATHEMATICS

_____A. Preschool/Introductory Math

_____1.   Basic counting skills to 10

_____2.

_____3.

_____4.

_____5.

_____6.

_____B. Primary Math

_____1.   Simple addition and subtraction skills

_____2.

_____3.

III. SPECIFIC INSTRUCTIONAL UNITS/KITS OF MATERIALS IN MATHEMATICS (Con't.)

    B. Primary Math (Con't.)

       ____ 4.

       ____ 5.

       ____ 6.

____ C. Basic Math Units

       ____ 1. Manipulation of sets of concrete objects

       ____ 2. Using the number line

       ____ 3. Two- and three-dimensional spatial relationships

       ____ 4.

       ____ 5.

       ____ 6.

       ____ 7.

____ D. Other Categories (Specify)_____

       ____ 1.

       ____ 2.

       ____ 3.

       ____ 4.

       ____ 5.

____ E. Other Categories (Specify)_____

       ____ 1.

       ____ 2.

       ____ 3.

       ____ 4.

       ____ 5.

____ F. Other Categories (Specify)_____

       ____ 1.

       ____ 2.

       ____ 3.

       ____ 4.

III. SPECIFIC INSTRUCTIONAL UNITS/KITS OF MATERIALS IN MATHEMATICS

_____ G. Other Categories (Specify)_____

    ___·1.

    ___2.

    ___3.

    ___4.

    ___5.

_____ H. Other Categories (Specify)_____

    ___1.

    ___2.

    ___3.

    ___4.

    ___5.

IV. SPECIFIC INSTRUCTIONAL AIDS (MANIPULATIVE/TACTILE AIDS, TEACHING MACHINES, CALCULATORS, ETC.)

Do you have suggestions for specific tangible aids or instructional materials in the following categories? If so, please comment or describe:

1. Reference materials

2. Models

IV. SPECIFIC INSTRUCTIONAL AIDS (Con't.)

   3. Independent study/self-insructional materials

   4. Skill kits

   5. Basic math concept units (e.g. spatial relationships)

   6. Charts

   7. Teaching machines/programed materials

IV. SPECIFIC INSTRUCTIONAL AIDS (Con't.)

8. Manipulative devices

Number-Blox with accompanying Book A & B   (Example of exponential blocks)
Attribute Blocks
Sorting materials
Cooking utensils
Graphing materials - physical objects
                       -butcher paper, felt pens, yarn, etc. for displaying
Variety of boxes for volume measurement
Variety of scales
Variety of timers, sand-times, clocks
Real money

9. Calculator/computer materials

Hand-held calculators   (Younger children should explore at random)

10. Supplementary materials

See list under IV. 8

11. Audio materials

The least amount of money should be spent here. Unless children do the threading of the machines, this is usually a very passive activity.

12. Student project materials, games, puzzles, etc.

The greatest variety possible. Examples: Mastermind, Foresight, Othello, Kalah, spatial relationship puzzles like LoonyLoop, Wff-n-Proof

13. Priorities suggested for future research and development pertaining to the APH Student Speech+ calculator

    / a. Development of (sequenced) activities with manipulative aids for introducing and applying the calculator across grade levels wherever the calculator can be utilized.

    / b. Development and evaluation of specialized aids, including over-lays or sheilds for learning the keyboard and key operations, to facilitate beginning student use of the calculator.

    ___ c. Preparation and inclusion of criterion tests to evaluate student progress in performing computational tasks using the calculator. Tests to include items for use with and without the calculator.

    / d. Materials to introduce the calculator to primary-elementary grade visually handicapped students.

    ___ e. Workbook practice materials which focus on computation for elementary grade students.

    / f. More advanced materials which emphasize problem solving for upper elementary, junior high, and secondary students.

    ___ g. Specialized supplementary materials to implement calculator usage in areas such as statistics and probability, algebra, trigonometry, and scientific applications.

    / h. Development of audio-tutorial materials for students who are able to use them.

    ___ i.

    ___ j. *In general, look for ways to involve students in physically active learning experiences in mathematics.*

    ___ k. "Keystrokes--Addition and Subtraction and Keystrokes--Multiplication and Division, available from Creative Publications."   Wheatley

    ___ l. "Materials from NCTM and Hewlett Packard."   JP
"EMC^2 materials from TI." Kessner

14. Can you suggest categories of new content material (e.g. statistics and probability) which will be included in the math curriculum within the next 3-5 years? Please note.

I believe that more and more research will become available that looks at the relationship between programs in movement and their relationship to the acquisition of mathematical concepts. I am personally involved in such research and the interest in the field is phenomenal.

"Think the statistics/probability strand will be given development." E.G.G.

"Content material appropriate for assistance using calculator and computer." E.G.G.

"Certainly statistics/probability." J.P.

"Increased emphasis on calculator." J.P.

"This strand (statistics and probability) should begin in grade one and build throughout the grades. I can show you good materials." G.H.W.

"Activities to develop spatial ability and imagery in mathematics learning." G.H.W.

"I see little change except for use of calculator. It will be the biggest change mechanism for the next 5 years." G.I.

"We are in a period of 'basic ideas.'" G.I.

"Estimation, statistics/probability, rounding, large numbers (greater than 10^8), and computer programming (for upper grades)." A.K.